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ABSTRACT

Through Monte Carlo procedures, three different techniques for estimating the parameter theta (proportion of the "shocks" remaining in the system) in the Integrated Moving Average (0,1,1) time-series model are compared in terms of (1) the accuracy of the estimates, (2) the independence of the estimates from the true value of theta, and (3) the independence of the estimates from a 'shift in level' in the time-series following an intervention. In the "usual" range for theta, the methods appear equally accurate. One produces complex estimates in special cases. Estimates are independent of the true value and changes in level. (Author)

The Estimation of Theta in the Integrated Moving Average Time-Series Model

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1. Background

The Integrated Moving Average (IMA) models for analysis of time series data have been increasingly useful in the behavioral sciences, including educational research. Specifically, these models are well-suited for testing hypotheses arising from interventions in either experimental or non-experimental situations; the researcher can compare a variable's pattern of behavior before the intervention has occurred with its behavior afterwards, and can do so without having to meet common assumptions of stochastic independence of observations (see Glass, Willson, and Gottman, 1975 for methods and examples.)

Of these models, the model IMA (0,1,1) is frequently identified as a good descriptor of sample time series data. This model has the form

$$(1.1) \qquad z_t - z_{t-1} = a_t - \theta a_{t-1}$$

where z_i = observation or datum recorded at time period i, a_i = random "shock" at time i, and θ(theta) = a fixed constant. It postulates (in words) that the difference between two consecutive observations is due to a random shock at the time of the current observation, minus (or plus, depending on the sign of θ) some fixed proportion (θ) of shock "left over" from the preceding observation.

The single parameter θ measures "carryover" of the influence of the random shocks; for reasons of mathematical stability, θ must be in the interval (-1,+1), and so may indeed be thought of as a proportion.

IMA (0,1,1) can be rearranged in various ways to incorporate parameters measuring patterns in the data, or changes in patterns coincident with interventions; such parameters may be used to measure

series level, change in level after intervention, series drift, or change in series drift after intervention.

For example, appropriate rearrangement of (1.1) yields

$$(1.2) \qquad z_t = L + (1-\theta)\sum_{i=1}^{t-1} a_i + a_t \quad ,$$

which expresses z as a sum (hence, <u>integrated</u> moving average) of previous and current random shocks; the parameter L has been added to indicated the "level" of the series previous to observation 1. A value of L may be estimated from the data, given a suitable value of θ; more typically, however, it is a <u>change</u> in series level that is of interest. By postulating (1.2) before a treatment event (or intervention) E occurs, and by postulating

$$(1.3) \qquad z_t = L + \delta + (1-\theta)\sum_{i=1}^{t-1} a_i + a_t$$

after E, one may estimate not only L, but estimate δ (change in series level at E) as well. Once again, this estimation requires a suitably accurate value of θ.

Other models may be derived, and parameters defined as needed. A transformation of the raw data and utilization of the general linear model permits least-squares estimates of these parameters of interest, along with appropriate tests of hypotheses using nothing more esoteric than Student's t-distribution (Glass, Willson, and Gottman, 1975, pp. 136 ff.); all such procedures, however, necessarily depend on the specific value of θ used. Since θ is itself generally unknown, some procedure must be used for finding the "appropriate" value.

Three such methods for "choosing" θ have been suggested. The first of these selects the value of θ which minimizes $\sum_{i=1}^{N} a_i^2$ in the general linear model $y = Xb + a$; here, y is a column vector of transformed data defined by $y_1 = z_1$ and $y_t = z_t - z_{t-1} + \theta y_{t-1}$ for $t>1$; X is the N x 2 "design" matrix whose (i,1)<u>th</u> entry is θ^{i-1}, and whose (i,2)<u>th</u> entry is 0 if $i \leq n_1$, and $\theta^{i-n_1} - 1$ if $i > n_1$ (here n_1 = number of time points preceding the intervention

E, and N = total number of time points in the series); b is the vector $\begin{bmatrix} L \\ \delta \end{bmatrix}$, and a is a column vector of random shocks (errors) a_i. The quantity $\sum_{i=1}^{n} a_i^2$ is easily computed as $(y - Xb)^T (y - Xb)$. This method yields the maximum likelihood estimate of theta. In what follows, we shall refer to this method as SSE or SSEMIN, for "Sum of Squared Errors, MINimized."

The second method is a Bayesian approach: we use the computed value of $S_a^2 = (y - Xb)^T (y - Xb)/(N - 2)$ to define the function $h(\theta|z) = |X^T X|^{-\frac{1}{2}} Sa^{-(N-1)}$, and choose θ such that h is maximized. This method assumes an "uninformed" prior distribution. Box and Tiao (1965, p. 189) give an explicit formula for h for the case of models (1.2) and (1.3). Hereafter we shall refer to this procedure as PD or PDMAX, for "Posterior Distribution MAXimization."

The third method merely solves for θ in the theoretical identity

(1.4) $\rho_1 = -\theta / (1 + \theta^2)$

(Box and Jenkins, 1970, p. 69), where ρ_1 is the lag-1 autocorrelation (which can easily be estimated from the data). We refer to this method as CØRR.

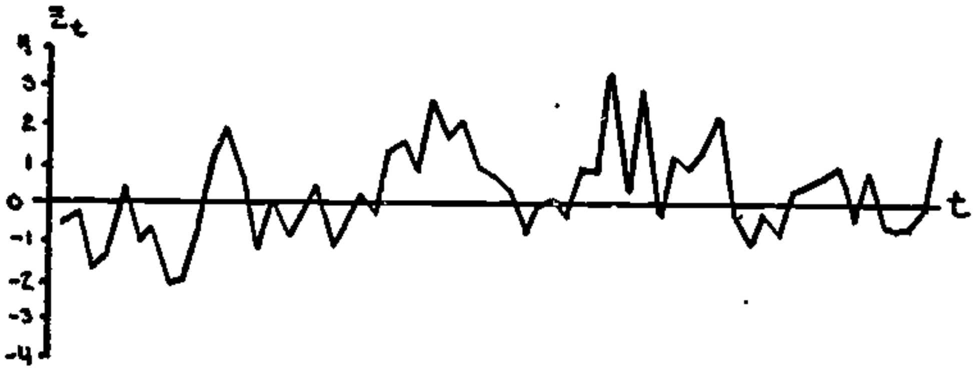
2. Objectives

No decision rule exists for "selecting" the "appropriate" value of theta. In fact, no procedures are available for determining whether one method should be preferable to the others. Although the values of theta produced by the three methods are frequently in close agreement, there are instances in which they may differ widely. Three examples will illustrate the potential difficulties.

Figures 1,2, and 3 represent time series generated from random numbers a_j and preassigned parameter values. In each case, an IMA (0,1,1) model equivalent to (1.2) and (1.3) was used to generate the series, with $n_1 = 30$, $N = 60$, $L = 0$, $\delta = 0$, and $\theta = .40$. The error terms were NID (0,1). The results are summarized below:

SERIES	SSEMIN θ	PDMAX θ	CORR θ	TRUE θ
1	.77	.56	.25	.40
2	.99	.99	.45	.40
3	.99	.31	undefined	.40

Series 1 is distinguished by complete disagreement between the three methods, with differences on the order of .2. In Series 2, SSEMIN and PDMAX have "topped out," producing estimates at or near the upper limit of permissible values of θ; note, however, that CORR has produced a good estimate of θ. Series 3 displays yet another "pathological" situation: SSEMIN has topped out, PDMAX appears normal, and CORR has produced a complex estimate of θ! (The latter circumstance occurs whenever $|\rho_1| > .5$) It should be noted here that these examples were not contrived; they appeared in the first 100 time series generated during the testing of the computer programs used in this study.



<u>Figure 1</u> A Time Series Defined by $z_t - z_{t-1} = a_t - .4a_{t-1}$, for which SSEMIN $\hat{\theta}$ = .77, PDMAX $\hat{\theta}$ = .56, and CORR $\hat{\theta}$ = .25. (Raw data values are given below.)

t	z_t	t	z_t	t	z_t	t	z_t
1	-.50955	16	-.82147	31	.35059	46	-.29940
2	-.34788	17	-.24159	32	-.70319	47	-.91187
3	-1.75570	18	.50792	33	-.03351	48	-.33964
4	-1.30331	19	-1.10159	34	.15259	49	-.75124
5	.39748	20	-.62420	35	-.33364	50	.36481
6	-1.05543	21	.17189	36	.90781	51	.52576
7	-.73269	22	-.27972	37	.90359	52	.73059
8	-2.10251	23	1.28653	38	3.39536	53	1.06632
9	-1.93148	24	1.58326	39	.43409	54	-.41533
10	-.67561	25	.81504	40	2.88648	55	.85683
11	1.04247	26	2.63036	41	-.27226	56	-.56898
12	1.94783	27	1.67359	42	1.29166	57	-.57721
13	.89106	28	2.04245	43	.94709	58	-.46467
14	-1.21484	29	.99347	44	1.25019	59	-.01620
15	.05537	30	.77547	45	2.20323	60	1.81171

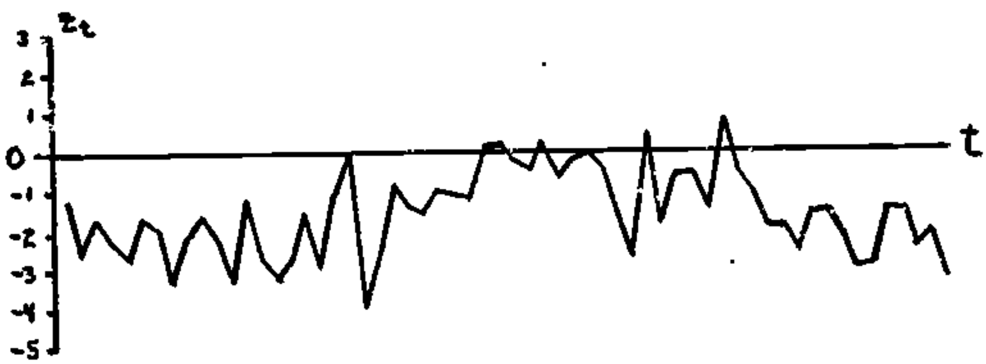


Figure 2 A Time Series Defined by $z_t - z_{t-1} = a_t - .4a_{t-1}$, for which SSEMIN $\hat{\theta} = .99$, PDMAX $\hat{\theta} = .99$, and CORR $\hat{\theta} = .45$. (Raw data values are given below.)

t	z_t	t	z_t	t	z_t	t	z_t
1	-1.20872	16	-2.65481	31	-.25791	46	-.53320
2	-2.61541	17	-1.55089	32	-.54480	47	-1.08796
3	-1.76947	18	-2.92075	33	.21269	48	-1.94553
4	-2.26526	19	-1.09186	34	-.74336	49	-1.97188
5	-2.74086	20	-.05491	35	-.33768	50	-2.54917
6	-1.69193	21	-3.96347	36	-.24049	51	-1.60747
7	-1.90799	22	-2.56271	37	-.49230	52	-1.56289
8	-3.29326	23	-.89612	38	-1.73034	53	-2.27819
9	-2.25422	24	-1.43146	39	-2.74358	54	-2.99438
10	-1.61278	25	-1.57890	40	.43743	55	-2.89742
11	-2.34021	26	-1.01972	41	-1.81990	56	-1.45638
12	-3.37741	27	-1.20197	42	-.72163	57	-1.50152
13	-1.19264	28	-1.25736	43	-.63091	58	-2.53836
14	-2.82437	29	.16511	44	-1.52007	59	-2.06113
15	-3.27598	30	.14939	45	.72893	60	-3.33544

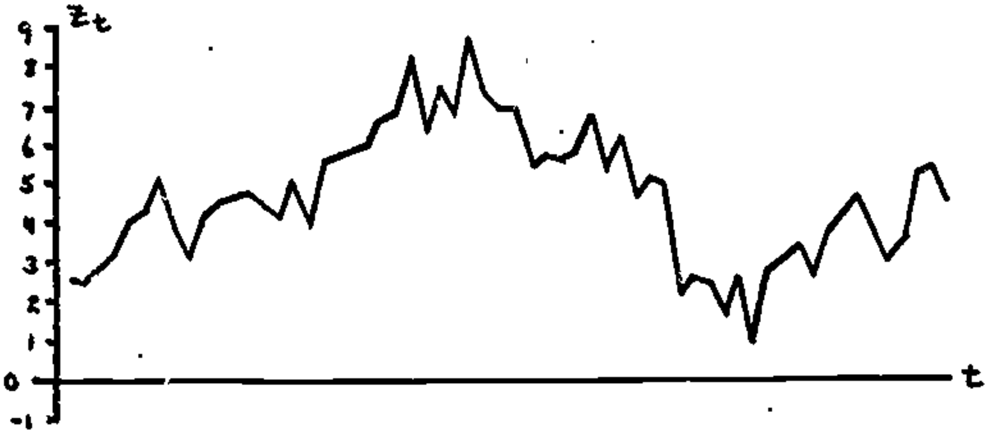


Figure 3 A Time Series Defined by $z_t - z_{t-1} = a_t - .4a_{t-1}$, for which SSEMIN $\hat{\theta}$ = .99, PDMAX $\hat{\theta}$ = .31, and CORR $\hat{\theta}$ is undefined. (Raw data values are given below.)

t	z_t	t	z_t	t	z_t	t	z_t
1	2.54168	16	5.04931	31	6.97965	46	2.64907
2	2.51421	17	3.86878	32	5.47227	47	.99328
3	2.82920	18	5.55951	33	5.75792	48	2.72786
4	3.18158	19	5.71685	34	5.67783	49	3.19561
5	4.01591	20	5.88405	35	5.84184	50	3.45374
6	4.32939	21	6.06740	36	6.73658	51	2.62422
7	5.07710	22	6.58521	37	5.32152	52	3.75585
8	3.88292	23	6.91787	38	6.16077	53	4.28520
9	3.07291	24	8.31332	39	4.65887	54	4.63624
10	4.14799	25	6.34903	40	5.23080	55	3.83178
11	4.54090	26	7.45182	41	4.96766	56	3.03398
12	4.63839	27	6.78753	42	2.20366	57	3.62824
13	4.68552	28	8.71404	43	2.61580	58	5.14878
14	4.44781	29	7.36904	44	2.49948	59	5.40953
15	4.08170	30	6.98560	45	1.70197	60	4.58758

Thus, we ask the following questions:

(1) How accurately do the three methods estimate theta?

(2) To what extent does each method's accuracy depend on the true value of theta?

(3) To what extent does the value of another parameter in the model (namely, a change in series level: δ) influence the accuracy of each method?

3. Method

"Monte Carlo" simulation techniques were deemed appropriate, and were utilized on the University of Minnesota's Control Data Cyber 74 computer.

Twenty populations of time series of the form shown in (1.2) and (1.3) were defined; ten for which theta was given a value of .99, .9, .7, .5, .3, .1, 0, -.3, -.5, and -.99, respectively, and delta was zero, and ten more with the same values of theta, and delta = .5. (More positive values than negative were used for theta because theta is nearly always positive in the real world.) For each of these 20 populations, 1000 sample series were generated; each of these series had $n_1 = 30$, $N = 60$, $L = 0$, and used random shocks a_i that were normal, independent, with mean 0 and variance 1. For each of the 20,000 sample series thus defined, theta was estimated from the data by the methods SSEMIN, PDMAX, and CORR; these numbers, plus the lag - 1 autocorrelation (referred to hereafter as LAG) were retained, and descriptive statistics computed.

For each preassigned value of theta, a Smirnov two-sample goodness-of-fit test was performed, comparing the distributions for which $\delta = 0$ with those for which $\delta = .5$. (Conover, 1971, pp. 309-314)

4. Results

Descriptive statistics produced by the 20 computer runs are displayed in Tables 1-5.

Table 1 shows that SSEMIN and PDMAX are comparably accurate over all values of θ tested; the means are within .025 of the true values of θ, except near the extremes, where differences of .09 or so can occur. The medians of SSEMIN and PDMAX are similarly accurate, and are generally better estimates near theta's extreme values. The modes reflect the topping-out or bottoming-out effect noted previously.

Table 2 shows all three methods to be of surprisingly consistent accuracy, in the sense that the distributions of $\hat{\theta}$ all have standard errors on the order of .01, independent of either θ or δ.

Table 3 reveals (as one might expect) that as the true value of θ deviates from 0 (the midpoint of its possible range of values) the distribution of estimates of θ provided by SSEMIN and PDMAX become less and less symmetric.

The evidence for CORR is somewhat less encouraging; although it is substantially easier to compute in practice than either SSEMIN or PDMAX, we see from Tables 1-3 that the behavior of its estimates is much less desirable than that of the other methods. Its mean $\hat{\theta}$ appears to be tolerably accurate only in the range 0 to .6 or so (albeit the most common real-life range for θ), though less so than the other methods. It is both "quicker" and "dirtier" than its companions.

CORR does not show a tendency toward skewness at extreme values of true theta; this lack of "sensitivity", as well as part of the method's general inaccuracy, can be attributed to the fact that a large portion of the distributions tested had lag - 1 autocorrelations (LAG here) that

Table 1: Measures of Central Tendency Computed for Various Chosen Values of Theta and Delta; Tabled Values are Estimates of Theta, Based on 1000 Computer-Generated Time Series.

		MEAN $\hat{\theta}$			MEDIAN $\hat{\theta}$			MODE $\hat{\theta}$		
TRUE THETA(θ)	TRUE DELTA(δ)	SSE	PD	CORR	SSE	PD	CORR	SSE	PD	CORR
-.99	.0	-.952	-.950	-.480	-.989	-.984	-.467	-.990	-.990	-.420
-.99	.5	-.924	-.904	-.481	-.957	-.911	-.475	-.990	-.990	-.440
-.5	.0	-.507	-.513	-.375	-.516	-.499	-.366	-.990	-.990	-.240
-.5	.5	-.517	-.525	-.385	-.526	-.511	-.371	-.990	-.990	-.370
-.3	.0	-.320	-.314	-.240	-.321	-.311	-.223	-.990	-.320	-.120
-.3	.5	-.314	-.309	-.232	-.308	-.297	-.217	-.990	-.990	-.120
.0	.0	-.006	-.009	.036	-.002	-.002	.034	-.030	-.030	.040
.0	.5	.007	.004	.051	.005	.004	.044	.050	.000	.020
.1	.0	.109	.115	.147	.118	.116	.135	.990	.120	.220
.1	.5	.095	.098	.130	.096	.095	.123	.990	.020	.170
.3	.0	.305	.305	.308	.302	.291	.299	.990	.260	.200
.3	.5	.317	.313	.302	.312	.300	.290	.990	.290	.250
.5	.0	.510	.514	.427	.523	.505	.416	.990	.990	.510
.5	.5	.524	.521	.426	.524	.506	.415	.990	.990	.410
.7	.0	.714	.717	.487	.745	.712	.471	.990	.990	.460
.7	.5	.716	.708	.486	.730	.701	.482	.990	.990	.630
.9	.0	.877	.890	.521	.963	.905	.516	.990	.990	.490
.9	.5	.881	.873	.519	.930	.882	.515	.990	.990	.510
.99	.0	.926	.945	.503	.989	.935	.499	.990	.990	.610
.99	.5	.902	.893	.529	.960	.912	.530	.990	.990	.520

Table 2: Measures of Variability Computed for Various Chosen Values of Theta and Delta; Tabled Values Refer to Estimates of Theta, Based on 1000 Computer-Generated Time Series.

		STD. ERROR $\hat{\theta}$			STD. DEV. $\hat{\theta}$			VARIANCE $\hat{\theta}$		
TRUE THETA(θ)	TRUE DELTA(δ)	SSE	PD	CORR	SSE	PD	CORR	SSE	PD	CORR
-.99	.0	.008	.004	.008	.256	.117	.198	.066	.014	.039
-.99	.5	.007	.003	.007	.224	.083	.186	.050	.007	.035
-.5	.0	.010	.008	.007	.329	.243	.200	.108	.059	.040
-.5	.5	.010	.007	.007	.320	.211	.202	.102	.045	.041
-.3	.0	.009	.007	.007	.287	.237	.204	.083	.056	.042
-.3	.5	.009	.008	.007	.294	.243	.213	.087	.059	.045
.0	.0	.009	.007	.006	.295	.235	.201	.087	.055	.040
.0	.5	.009	.007	.007	.276	.212	.212	.076	.045	.045
.1	.0	.010	.008	.007	.304	.245	.210	.092	.060	.044
.1	.5	.009	.007	.007	.300	.233	.200	.090	.054	.043
.3	.0	.009	.007	.007	.292	.232	.207	.085	.054	.043
.3	.5	.010	.008	.007	.303	.250	.208	.092	.063	.043
.5	.0	.010	.007	.007	.311	.209	.193	.096	.044	.037
.5	.5	.009	.007	.007	.298	.231	.200	.089	.053	.040
.7	.0	.011	.007	.008	.335	.213	.185	.113	.045	.034
.7	.5	.010	.007	.008	.305	.210	.185	.093	.044	.034
.9	.0	.011	.005	.008	.346	.166	.186	.119	.027	.035
.9	.5	.009	.005	.008	.278	.168	.188	.077	.028	.035
.99	.0	.011	.005	.008	.338	.148	.183	.114	.022	.033
.99	.5	.010	.006	.008	.312	.188	.185	.097	.035	.034

Table 3: Skew and Kurtosis Computed for Various Chosen Values of Theta and Delta; Tabled Values Refer to Estimates of Theta, Based on 1000 Computer-Generated Time Series.

		SKEW $\hat{\theta}$			KURTOSIS $\hat{\theta}$		
TRUE THETA(θ)	TRUE DELTA(δ)	SSE	PD	CORR	SSE	PD	CORR
-.99	.0	7.416	13.617	-.122	53.269	219.907	-.363
-.99	.5	8.081	12.348	.091	65.998	275.592	-.439
-.5	.0	2.707	2.291	-.275	11.221	15.817	-.037
-.5	.5	2.739	1.589	-.202	11.901	14.923	-.160
-.3	.0	1.491	1.100	-.284	8.803	11.066	.480
-.3	.5	1.388	.897	-.382	8.079	9.845	.357
.0	.0	-.107	-.434	.190	6.085	9.378	1.046
.0	.5	.021	-.145	.245	6.523	9.615	.628
.1	.0	-.729	-.389	.293	6.152	9.099	.565
.1	.5	-.402	.006	-.044	5.751	8.743	.861
.3	.0	-1.491	-.734	.343	8.467	10.183	.415
.3	.5	-1.481	-1.042	.148	8.022	10.172	-.070
.5	.0	-2.846	-1.955	.194	12.837	17.527	-.195
.5	.5	-2.669	-1.970	.242	12.972	15.149	-.193
.7	.0	-3.974	-4.434	.166	17.795	35.009	-.247
.7	.5	-4.074	-4.009	.089	20.560	31.704	-.590
.9	.0	-5.041	-8.822	.056	24.342	97.048	-.432
.9	.5	-6.127	-8.276	.015	28.436	89.058	-.298
.99	.0	-5.480	-11.911	.097	28.140	144.187	-.465
.99	.5	-5.732	-9.072	-.135	32.098	87.867	-.272

fell out of range (see Table 5). Without this truncation, the LAG estimates provided good estimates of the true lag - 1 autocorrelation (which can then be transformed to theta via (1.4)). Summary statistics of these distributions of nontruncated LAG estimates appear in Table 4.

(Table 5 also displays percentages of the samples tested for which SSEMIN and/or PDMAX topped- or bottomed -out. This gives us a rough idea of the expected frequency of these situations.)

Finally, we note from Table 6 that most of the distributions generated by SSEMIN, PDMAX, and LAG showed a theoretical dependence on the value of δ, whereas those distributions generated by CORR showed little dependence on δ. The test statistic being evaluated is the longest vertical distance between the cumulative density functions of the two sample distributions under scrutiny (Conover, 1971, p. 310).

5. Conclusions

SSEMIN and PDMAX appear to estimate theta adequately in all ranges of true theta. CORR is less accurate, especially outside the range .0 to .6, although the lag - 1 autocorrelations (LAG) of samples are good estimators of the true autocorrelation ρ_1. Practical problems in using each method include the very real possibility that an estimator will "top out" or "bottom out", or, in the case of CORR, not exist.

Table 4: Summary Statistics Computed for Various Chosen Values of Theta and Delta: Tabled Values Refer to Estimates of the Lag-1 Autocorrelation, Based on 1000 Computer-Generated Time Series.

TRUE THETA(θ)/ TRUE LAG-1 CORRELATION(R)	TRUE DELTA(δ)	CENTRAL TENDENCY			VARIABILITY			HIGHER MOMENTS	
		MEAN	MEDIAN	MODE	STD. ERROR	STD. DEV.	VARIANCE	SKEW	KURTOSIS
-.99 / .499	.0	.434	.448	.510	.004	.136	.018	-.386	.246
-.99 / .499	.5	.452	.457	.370	.004	.137	.019	-.328	-.134
-.5 / .400	.0	.342	.348	.360	.005	.151	.023	-.294	-.006
-.5 / .400	.5	.351	.360	.430	.005	.151	.023	-.378	.011
-.3 / .275	.0	.216	.221	.190	.005	.165	.027	-.182	-.080
-.3 / .275	.5	.207	.214	.260	.005	.171	.029	-.230	-.258
.0 / .0	.0	-.030	-.033	-.040	.006	.179	.032	.069	-.130
.0 / .0	.5	-.043	-.044	-.080	.006	.190	.036	.073	-.010
.1 /-.099	.0	-.132	-.135	-.210	.006	.179	.032	.169	-.319
.1 /-.099	.5	-.120	-.123	-.170	.006	.182	.033	.234	-.016
.3 /-.275	.0	-.279	-.292	-.340	.005	.162	.026	.274	.117
.3 /-.275	.5	-.280	-.291	-.250	.005	.170	.029	.318	-.043
.5 /-.400	.0	-.399	-.404	-.390	.005	.146	.021	.267	-.007
.5 /-.400	.5	-.392	-.400	-.410	.005	.145	.021	.347	.002
.7 /-.470	.0	-.461	-.466	-.550	.004	.136	.018	.314	.030
.7 /-.470	.5	-.455	-.461	-.450	.004	.134	.018	.301	-.211
.9 /-.497	.0	-.480	-.484	-.480	.004	.131	.017	.260	.175
.9 /-.497	.5	-.478	-.484	-.480	.004	.130	.017	.410	.266
.99 /-.499	.0	-.482	-.490	-.560	.004	.132	.017	.307	-.130
.99 /-.499	.5	-.491	-.500	-.520	.004	.127	.016	.495	.491

Table 5: Percentage of 1000 Computer-Generated Time Series Judged "Out of Range." For SSE and PD, BOT = % Distributions with $\hat{\theta} \leq -.99$, and TOP = % Distributions with $\hat{\theta} \geq .99$; for LAG, BOT = % Distributions with $R_1 \leq -.5$, and TOP = % Distributions with $R_1 \geq .5$

TRUE THETA(θ)/ TRUE LAG-1 CORRELATION(P)	TRUE DELTA(δ)	SSE			PD			LAG		
		BOT	MID	TOP	BOT	MID	TOP	BOT	MID	TOP
-.99 / .499	.0	85.7	12.6	1.7	49.6	50.2	0.2	0.0	68.3	31.7
-.99 / .499	.5	32.8	65.9	1.3	11.9	88.0	0.1	0.0	62.7	37.3
-.5 / .400	.0	9.7	87.1	3.2	7.5	91.5	1.0	0.0	84.0	16.0
-.5 / .400	.5	9.1	88.0	2.9	6.4	93.2	0.4	0.0	84.2	15.8
-.3 / .275	.0	5.6	92.1	2.3	3.3	95.6	1.1	0.0	96.8	3.2
-.3 / .275	.5	5.9	91.7	2.4	3.9	95.3	0.8	0.0	97.3	2.7
.0 / .0	.0	3.7	93.0	3.3	1.5	97.4	1.1	0.2	99.5	0.[illegible]
.0 / .0	.5	2.7	94.4	2.9	1.0	97.9	1.1	0.4	99.3	0.[illegible]
.1 /-.099	.0	3.6	92.4	4.0	1.2	96.2	2.6	1.3	98.7	0.0
.1 /-.099	.5	3.1	92.9	4.0	0.9	97.4	1.7	1.3	98.7	0.0
.3 /-.275	.0	2.5	92.1	5.4	0.6	95.8	3.6	8.5	91.5	0.0
.3 /-.275	.5	2.7	91.0	6.3	1.1	94.9	4.0	9.9	90.1	0.[illegible]
.5 /-.400	.0	2.8	89.0	8.2	0.4	94.3	5.3	26.9	73.1	0.[illegible]
.5 /-.400	.5	2.3	88.6	9.1	0.8	92.7	6.5	26.0	74.0	0.[illegible]
.7 /-.470	.0	3.1	79.7	17.2	0.8	86.6	12.6	42.5	57.5	0.[illegible]
.7 /-.470	.5	2.4	80.1	17.5	0.6	86.1	13.3	41.0	59.0	0.[illegible]
.9 /-.497	.0	3.2	53.5	43.3	0.6	71.6	27.8	47.0	53.0	0.[illegible]
.9 /-.497	.5	2.0	61.8	36.2	0.6	75.5	23.9	47.0	53.0	0.[illegible]
.99 /-.499	.0	3.0	10.6	86.4	0.4	48.7	50.9	48.6	51.4	0.[illegible]
.99 /-.499	.5	2.6	64.0	33.4	0.8	87.1	12.1	51.9	48.1	0.[illegible]

Table 6: Smirnov Two-Sample Test Statistics, Comparing $\hat{\theta}$ Distributions with δ = 0 to those with = .5. * = Significant at alpha = .05, ** = significant at alpha = .01; all tests are 2-tailed.

TRUE THETA (θ)	SSEMIN	PDMAX	CORR	LAG
-.99	.895**	.536**	.057	.100**
-.5	.098**	.080**	.054	.068*
-.3	.064*	.067*	.046	.050
.0	.072*	.078**	.053	.057
.1	.103**	.105**	.071	.071*
.3	.065*	.068*	.048	.056
.5	.091**	.076**	.049	.051
.7	.175**	.133**	.051	.062*
.9	.433**	.278**	.042	.043
.99	.864**	.525**	.095*	.075**

Each estimation method is consistently accurate, in the sense that if the specific estimate $\hat{\theta}$ is thought of as a sample chosen from a theoretical distribution of θ, then the standard error of the estimate is likely to be less than .01.

Although the presence of a change in level has little practical impact on the estimated value of θ (cf. Table 1), other investigation reveals (Table 6) that the value of δ does change the nature of the theoretical distribution of estimates of theta.

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